Novel Topology Optimization
Based on On-Off Method and Level Set Approach

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Outline of the presentation

I. Background and purpose
II. Present method
III. Optimization Results
IV. Conclusions
Background

- Shape optimization plays an important role in the development of electromagnetic devices.
- There are two approaches for shape optimizations, namely, parameter and topology optimizations.

Parameter optimization

\[ x_1 \sim x_3 : \text{Material parameter} \]
Background

- Shape optimization plays an important role in the development of electromagnetic devices.
- There are two approaches for shape optimizations, namely, parameter and topology optimizations.

**Parameter optimization**

- Device shape are represented with design parameters
- Optimization is conducted by changing the parameters.

**Topology optimization**

- This method seeks for the optimum solutions directly varying the material shape without design parameters.

Dependence on experience and knowledge of engineers

Find novel shape
In the topology optimization on-off and level-set methods are widely used.

- Genetic Algorithm (GA) is widely employed for optimization process.
- Material shapes are expressed as binary pixel images
Background

- In the topology optimization on-off and level-set methods are widely used.

On-Off Method

- Genetic Algorithm (GA) is widely employed for optimization process.
- Material shapes are expressed as binary pixel images.

- We may obtain complicated shape because of huge search spaces.
In the topology optimization on-off and level-set methods are widely used.

- Material boundaries are expressed with level set function.
- We can have smooth boundaries and non-porous material region.
- This tends to fail into local optima because optimization is conducted based on gradient method.
Purpose

- Present Method

First step is the global search.
- GA has good performance for the global search
- One solution is selected

<table>
<thead>
<tr>
<th>Global Search</th>
<th>Genetic Algorithm</th>
</tr>
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<tbody>
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<td>On-Off Method</td>
<td></td>
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- Higher fitness
- Smooth boundaries
- Non-porous material regions
Purpose

- Present Method

First step is the global search.
- GA has good performance for the global search
- One solution is selected

Second step is the local search,
- The solution improved by level set method
- Smooth boundaries and non-porous material region

![Diagram]

Global Search
- On-Off Method
- Genetic Algorithm

Local Search
- Level Set Method
- Gradient Method

- Higher fitness
- Smooth boundaries
- non-porous material regions
Outline of the present method

Global Search with GA

Generations
0
1

Local Search Based on Level Set approach

Steps
0
1
n

Resultant shape is expressed by the level set function.
Global search method – On-Off Method –

- In order to suppress computational time, the micro genetic algorithms (μGA) is employed for optimization [1].
- To eliminate high frequency component, we applied the averaging filter for smoothing.

Local search method - Level Set Method -

- Material shape is expressed in terms of the level set functions.
- The level set functions are defined on each node.
- The level set function of any point in each element calculates by interpolating.

- $D$: Design region
- $\Omega$: Material region
- $\partial \Omega$: Material boundary
- $x$: Point vector in $D$

$$\begin{align*}
\phi(x) = \begin{cases} 
> 0 & (x \in \Omega) \\
= 0 & (x \in \partial \Omega \cap D) \\
< 0 & (x \in D \setminus \Omega)
\end{cases}
\end{align*}$$

$\phi_i$: Level set function on each point
Local search method - Level Set Method -

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Distance function from material boundary

\[
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= 0 & (x \in \partial \Omega \cap D) \\
< 0 & (x \in D \setminus \Omega) 
\end{cases}
\]

\(\phi_i\): Level set function on each point
Level-Set method – Distance function –

- Level set function is defined by

\[
\phi(x) = \begin{cases} 
  d(x, \partial \Omega) & x \in \Omega \\
  0 & x \in \partial \Omega \\
  -d(x, \partial \Omega) & x \notin \Omega
\end{cases}
\]

where \( d \) denotes the shortest distance between \( x \) and boundary.

- The value of level-set function \( \phi \) is evaluated by

\[
\phi(x) = \min_{y \in \partial \Omega} d(x, y)
\]
Level-Set method – In the optimization –

- Material shapes are expressed with using level-set function and optimization is conducted by changing them.
- Level-set function is updated to reduce the value of objective function as follows:

\[
\phi_i^{n+1}(x) = \phi_i^n(x) + V_N
\]

\[
V_N = -\frac{df}{d \phi_i}
\]

\[
\frac{df}{d \phi_i} = \frac{\partial f}{\partial \phi_i} + \frac{\partial f}{\partial A} \cdot \frac{\partial A}{\partial \phi}
\]

- It is difficult to evaluate.

\( f \) : objective function
\( n \) : Iteration of optimization
\( V_N \) : update descent of the level-set functions
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\]

- In order to evaluate the gradient, adjoint variable method is employed.

\(f\) : objective function
\(n\) : Iteration of optimization
\(V_N\) : update descent of the level-set functions
Adjoint variable method

- Differentiate $f$ with respect to level-set function
  
  a. Modified objective function defined by (1)
  
  b. Differentiation of Eqn. (1) with respect to $\phi_i$ leads to (2)
  
  c. Update the level-set function using $V_N$

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\[
\hat{f} = f + z^T(KA-b) \quad (1)
\]
Adjoint variable method

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V_N = -\frac{df}{d\phi_i}
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\[
\hat{f} = f + z^T(KA - b)
\]

\( f \approx \hat{f} \) if \( A \) exactly satisfies \( KA = b \)
Adjoint variable method

- Differentiate $f$ with respect to level-set function
  
  \[ \phi_i^{n+1}(x) = \phi_i^n(x) + V_N \]
  
  \[ V_N = -\frac{df}{d\phi_i} \]

\[ f = f + z^T (KA - b) \] (1)

\[ \frac{df}{d\phi_i} = \frac{\partial f}{\partial \phi_i} + z^T \frac{\partial K}{\partial \phi_i} A + \left(z^T K + \frac{\partial f}{\partial A}\right) \frac{dA}{d\phi_i} \] (2)

In order to avoid evaluating this
Adjoint variable method

- Differentiate $f$ with respect to level-set function
  
  a. Modified objective function defined by (1)
  
  b. Differentiation of Eqn. (1) with respect to $\phi_i$ leads to (2)
  
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V_N = -\frac{df}{d\phi_i}
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Adjoint variable method

● Differentiate $f$ with respect to level-set function

\[
\begin{align*}
\text{a. Modified objective function defined by (1)} & \\
\text{b. Differentiation of Eqn. (1) with respect to } \phi_i \text{ leads to (2)} & \\
\text{c. Update the level-set function using } V_N
\end{align*}
\]

\[
\phi_{i}^{n+1}(x) = \phi_{i}^{n}(x) + V_N
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V_N = -\frac{df}{d\phi_i}
\]
Numerical example 1 - IPM-Motor -

- The purpose of this optimization is to maximize the torque average and minimize the torque ripple.
- Shape of the flux barrier in the rotor is optimized.

**Optimization problem**

\[ F = -T_{AVG} + W * T_{rip} \rightarrow \text{Min.} \]

where

\[ T_{rip} = \frac{T_{\text{max}} - T_{\text{min}}}{T_{AVG}} \]

- \(T_{AVG}\) : Torque average [Nm]
- \(T_{rip}\) : Torque ripple
- \(W\) : Weighting coefficient

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IPM–Motor – Analysis conditions –

- Rotation speed (rpm) 3000
- Armature current (A) 600
- Phase of current (degree) 20
- Residual flux density of PM (T) 1.0
- Width of teeth (mm) 3.3
- Length of Coil (mm) 25.9
- Thickness of PM (mm) 2.5
- Width of PM (mm) 21

✓ Computational time : 10 [h]
✓ Number of unknown in FE analysis : about 2,000

Computational environment
- CPU : Xeon X5660 (6-Core 2.8GHz, 6 × 256KB+12MB, 1333MHz) × 2
- Main memory : 12GByte
Optimization results

On-Off method

<table>
<thead>
<tr>
<th>Torque average (Nm)</th>
<th>5.280</th>
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<tbody>
<tr>
<td>Torque ripple</td>
<td>0.184</td>
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<td>Objective function</td>
<td>-0.806</td>
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On-Off + Level Set method

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Optimization results

On-Off method

\[ T_{\text{max}} - T_{\text{min}} \]

Present method

\[ T_{\text{max}} - T_{\text{min}} \]

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Optimization results – Flux distribution –

<table>
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<th>Torque average (Nm)</th>
<th>Torque ripple</th>
<th>Objective function</th>
</tr>
</thead>
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<tr>
<td>On-Off method</td>
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<td>-0.806</td>
</tr>
<tr>
<td>On-Off + Level Set method</td>
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<td>0.112</td>
<td>-1.018</td>
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F uphill grid pattern

On–Off method

On–Off + Level Set method
Optimization results – Flux distribution –

- Non Flux Barrier
- Flux Barrier

Due to the flux barriers, magnetic flux goes to the rotor surface.
**Numerical example 2 - Magnetic shield -**

- The present method is applied to magnetic shield model shown in figure.
- The purpose of this optimization is to minimize the flux density in Evaluated region and core volume created in design region.

**Optimization Problem**

\[
F(\phi) = W_M \frac{|B|_{\text{average}}}{10^{-5}} + \frac{S}{S_{\text{design}}} \rightarrow \text{Min.}
\]

- \(W_M\): weighting coefficient
- \(S\): volume of the core
- \(S_{\text{design}}\): volume of the design region
- \(B\): flux density of the evaluated region

![Diagram of magnetic shield model with optimization problem formulation and numerical example](image)
Numerical example 2 – Magnetic shield –

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[Diagram showing a 2D section of a magnetic shield with labels for the evaluated region, design region, and a coil with 100[A \cdot \text{turn}].]
### Magnetic shield - Optimization parameter -

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of elements in design region</td>
<td>2,488</td>
</tr>
<tr>
<td>Number of elements in analysis region</td>
<td>5,052</td>
</tr>
<tr>
<td>Generation of global search ($\mu$GA)</td>
<td>200</td>
</tr>
<tr>
<td>Generation of local search (Level Set)</td>
<td>200</td>
</tr>
<tr>
<td>Weighting coefficient: $W_M$</td>
<td>0.2</td>
</tr>
</tbody>
</table>

- ✓ Computational time: 2[h]
- ✓ Number of unknown in FE analysis: about

**Computational environment**

- CPU: Xeon X5660 (6-Core 2.8GHz, 6 × 256KB+12MB, 1333MHz) × 2
- Main memory: 12GByte
Magnetic shield

| Method                      | $|B|_{\text{average}}/10^{-5}$ | Volume of the core (cm$^2$) | Objective function |
|-----------------------------|------------------------------|-----------------------------|--------------------|
| On-Off method              | 0.124                        | 2.736                       | 0.0678             |
| On-Off + Level Set method  | 0.0990                       | 2.776                       | 0.0631             |

$|B|_{\text{average}}/10^{-5}$ indicates the average magnetic field strength. The volume of the core is given in cm$^2$. The objective function values reflect the performance of the magnetic shield methods.
Magnetic shield

Global search
Magnetic shield

Local search
Magnetic shield

Objective Function

\[ F(\phi) = W_M \left( \frac{|B|_{\text{average}}}{10^{-5}} \right) + \frac{S}{S_{\text{design}}} \]

- \( W_M = 0.8 \)
- \( W_M = 0.4 \)
- \( W_M = 0.3 \)
- \( W_M = 0.2 \)
Magnetic shield

\[ W_M = 0.8 \]

\[ W_M = 0.4 \]

\[ W_M = 0.3 \]

\[ W_M = 0.2 \]

Objective Function

\[ F(\phi) = W_M \frac{|B|_{\text{average}}}{10^{-5}} + \frac{S}{S_{\text{design}}} \]
**Magnetic shield \((W_M=0.4)\)**

### On-Off method

| \(|B|_{\text{average}}/10^{-5}\) | 0.0727 |
|---------------------------------|--------|
| Volume of the core (cm\(^2\))  | 5.864  |
| Objective function              | 0.121  |

### On-Off + Level Set method

| \(|B|_{\text{average}}/10^{-5}\) | 0.0541 |
|---------------------------------|--------|
| Volume of the core (cm\(^2\))  | 6.016  |
| Objective function              | 0.116  |
Magnetic shield – Consideration of Branch

- Non protuberance
- A protuberance

- Due to protuberance occurs from out shield, flux goes to outside of the shield.
Conclusions

- We present a new topology optimization method which is based on the on-off and level set methods.
- In order to test this method, it is applied to numerical examples.
- The results show the present method can effectively find optimal solutions which have better performances.

Future works

- Applied to the 3-dimensional problems and other devices
- Introduce the multi-objective GA